

FCD COMPUTER PROGRAMS FOR AREA CALCULATION

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ABSTRACT

The methods of computing area from maps are classified under two categories: (1) bounded area, and (2) weighted sub-areas. Pioneering methods of planimeter, coordinate squares, triangular rule, trapezoidal rule, Simpson's rule, double-meridian-distance (DMD), coordinates, and digitizer are compared with the relatively easy methods of weighing technique and finite segments, and also with a recently developed Monte Carlo process.

The bounded area which is used extensively in computing area of land use can be obtained by any one of three methods: graphical, arithmetical and computer. The graphical method includes four techniques: weighing, planimeter, coordinates squares and digitizer. The accuracy of this graphical method is highly dependent upon the skill of the analyst and the correctness of instruments. The arithmetical method involves three rules: triangular, trapezoidal, and Simpson. The accuracy of this arithmetical technique is dependent upon the number of offsets used to divide the entire region. If a region is of very irregular shape and of large size, then this arithmetical method is very tedious and cumbersome to use. The techniques of double-meridian-distance, coordinates, finite segments and Monte Carlo are introduced by using a computerized procedure. The signs of latitudes, departures, and starting point are easily confused in the methods of DMD and coordinates. Finite segments can overcome some limitations such as a figure having some inactive areas within the figure, but it is also strongly dependent upon the direction used to select the boundary nodes. Monte Carlo is only a method which is independent of the direction and can be used in any complicated form of figure.

The weighted sub-area which is used widely in computing the areas weighted to a measurement station can be obtained by any one of the four techniques: simple arithmetic method, Thiessen method, stratified method and modified Monte Carlo method; however, the modified Monte Carlo method can be simulated directly by the computerized procedure and also has been applied successfully to compute the weighted area to each measured station.

Finally, from this study it can be concluded that the weighing technique is relatively easy to use in the laboratory; the finite segments method is a quick computerized procedure when the direction of selecting boundary points are correctly performed; and the Monte Carlo process is more applicable and powerful than other computerized methods because this method is not only independent of the direction used to select the boundary nodes but also applicable to any complicated form of figures with arbitrary signs of latitudes, departures and starting points.

INTRODUCTION

The computation of an area of land is most frequently used in several fields of water resource systems. Area calculations of interest to water resource workers include land use estimation, water quality, ecological systems, stratified sampling programs, etc. Pioneering methods used in computing area from maps, (hereafter the area computations are only referred to the area computed from the maps), are those of planimeter, coordinate squares, triangular rule, trapezoidal rule, Simpson's rule, double-meridian-distance (DMD), and coordinates (Brinker, 1969). Recently, a digitizer was also used (SAC, 1972). However, if an area is very irregular in shape and/or is of large size or has inactive areas within its boundaries, then the previous methods are very difficult to adapt to a computerized procedure. Hence, there is a need to introduce some other practical methods from which the area can be calculated directly, either by the computerized procedure or by a weighing machine.

OBJECTIVES

The purposes of this study are:

- (1) To discuss the existing methods used in computing areas;
- (2) To introduce two relatively easy methods of weighing techniques and finite segments, and a recently developed Monte Carlo process, together with their ranges of application, and
- (3) To compare the advantages and disadvantages of each method.

DESCRIPTION OF METHODS

The methods of computing area from maps may be classified under two categories: (1) bounded area and (2) weighted to sub-areas.

BOUNDED AREA

The bounded area has been widely used in determining the area of land included within the boundaries. The area to be measured can be obtained by any one of the following methods:

- I. Graphical Methods: The area is measured by using instruments such as: planimeter, coordinate squares counter, digitizer and weighing machine.
 - a. Weighing Method: A weighing machine used in a laboratory can be employed to obtain the area. Three procedures are involved: First, a ratio between a known area and weight must be procured from a control unit. Second, the weight of an unknown area is weighed. Third, multiplying the weight of an unknown area by the ratio which is obtained in the first procedure gives the area of an unknown figure. The nonuniform thickness of the paper and changing humidity can affect significantly the accuracy of measurement.
 - b. Planimeter: The planimeter is the commonest way of checking the area of a figure. It is a small instrument consisting of an arm, carrying a tracing point, which is moved over the outline of the figure to be computed. Poor setting of the planimeter scale bar, and failure to check the scale constant by tracing a known area, can cause an error of measurement.
 - c. Coordinate Squares: The figure is marked off in squares of unit area. The number of complete unit squares included is counted, and the sum of the partial units are also estimated. A transparent paper marked in squares to some scale is placed over the figure and the number of squares and partial units counted. The number of squares can also be counted by a mechanical dot counter or a transparent paper dotter.

Using coordinate squares which are too large makes it difficult to estimate the partial blocks and could cause an error in computation.

- d. Digitizer: A digitizer is a device used to convert information in graphic form into numerical intelligence suitable for processing, recording, or transmission on a digital data system. After the (x, y) coordinates of each point on the outline of the figure is recorded, the size of the plane area also can be computed. This method is highly dependent upon the direction used to digitize the boundary coordinates.

II. Arithmetic Methods: A figure can be divided into geometrical shapes (triangles, trapezoids, and rectangles), and the following rules can be used to compute the area:

- a. Triangular Rule: A figure may be divided into simple triangles. The area can be computed by the formula:

$$\text{Area} = \sqrt{S(S-a)(S-b)(S-c)} \quad (1)$$

where

a, b, and c are the sides of the triangle

$$S = 0.5(a + b + c) \quad (2)$$

Another formula is used when an angle between two sides is known,

$$\text{Area} = 0.5 a b \sin C \quad (3)$$

where C is the angle included between two sides a and b.

- b. Trapezoidal Rule: If the figure is considered as made up of a series of trapezoids, all having the same base, the area can be determined based on the formula

$$\text{Area} = \frac{d}{2} (h_a + 2 \sum h + h_b) \quad (4)$$

where

d = common distance between offsets and

h_a , h_b , and h = first, last and intermediate offsets.

- c. Simpson's Rule: For generally parabolic areas, Simpson's one-third rule as follows is applicable to obtain the size of the plane area.

$$\text{Area} = \frac{d}{3} (h_a + 2 \sum h_{\text{odd}} + 4 \sum h_{\text{even}} + h_b) \quad (5)$$

where

d = common distance between offsets,

h_a, h_b, h_{odd} and h_{even} = first, last, odd and even offsets.

Furthermore, if the figure is very irregular in shape, then the arithmetical method is very difficult to adapt by a computerized procedure.

Therefore, the following computerized methods are introduced:

- III. Computer Methods: The following four methods can be performed by the computerized procedure.

- a. Double-Meridian-Distance (DMD): The DMD of a traverse line is twice the distance from a meridian through the most westerly station to the middle point of the line. The double areas of all of the trapezoids may now be found by multiplying each DMD by the adjusted latitude of that side. The area obtained by plus latitudes and minus latitudes should be considered as a positive and negative area, respectively. The sum of these double areas will be double the area of the figures to be measured. The disadvantages of this DMD method is that the signs of DMD's, latitudes, departures, or areas are easily confused.
- b. Coordinates: The area is equal to one-half the sum of the products obtained by each x-coordinate by the difference between the adjacent y-coordinate, taken in the same order around the figure. Similar to the DMD method, the signs of coordinates, latitudes, departures, or areas are easily confused.

If the signs and starting point are recorded carefully and without any inactive areas within the figure, the above two methods can be used quite satisfactorily by using a computerized procedure; however, in practical application the above methods can accrue difficulty such as an inactive area included within the figure. Therefore, the following two methods which can overcome the limitations are introduced.

- c. Finite Segments: The boundary of a figure is defined by a series of linear segments between node points. The clockwise or counter-clockwise direction used to record (x, y) coordinates is dependent upon the figure either to be active or inactive, respectively. The area is obtained by the following formula:

$$\text{Area} = \sum_{i=1}^{M-1} (X_{i+1} - X_i) \frac{Y_{i+1} + Y_i}{2} + (X_1 - X_M) \frac{Y_1 + Y_M}{2} \quad (6)$$

where X_i, Y_i = coordinates of boundary nodes;

i = boundary segment index; and

M = total number of segments.

- d. Monte Carlo Method: The Monte Carlo method is a procedure which takes advantage of the high speed of an electronic computer in solving complex problems in physical and mathematical fields. Monte Carlo applications in the field of science and engineering are summarized in books by Hammersley and Handscombe (1964) and Shrieder (1967). This method involves enclosing the figure to be measured within a rectangular area and by generating random numbers choosing points randomly distributed throughout this rectangle. The proportion of these points falling within the figured area is, in the limit, the proportion of the rectangular area contained within the figure. But, in practical

application, the previous method has some limitations which were removed by Shih and Hamrick (1974).

i. To Determine Whether a Random Point is Within or Without the Watershed:

Shih and Hamrick (1974) developed an alternative test based on the principle that for any completely bounded region, a radial line constructed in any direction from a given point must cross the boundary an odd number of times if the point is located within the region or an even number of times

if without (assuming zero to be an even number). This test is ambiguous only in the case of node points (the intersections of straight line segments representing the figure boundary).

They also developed a second rule with a computerized technique for solving the ambiguity that exists when the radial line penetrates the boundary at the node point.

ii. Procedure of Computation:

Based on the technique described in the previous section, the following procedures are used to obtain an analog of any shape of figure.

- (1) Enclose the irregularly shaped boundary with a rectangle whose coordinates are also recorded.
- (2) Read the X and Y coordinates of the boundary segments.
- (3) Compute the weighting factor of each boundary node according to Rule 2 (Shih and Hamrick, 1974).
- (4) Generate random points with uniform probability over the enclosing rectangle.

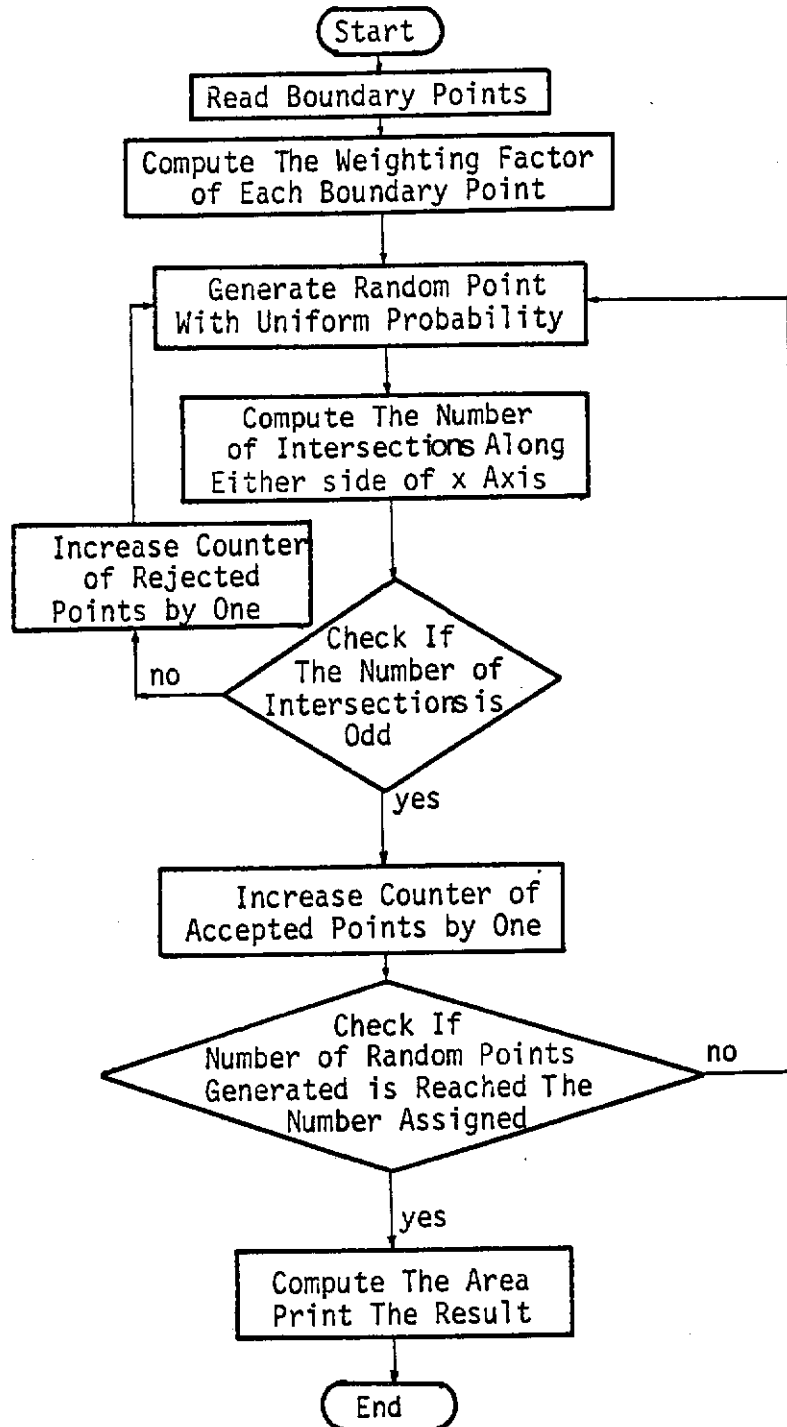
- (5) Draw an imaginary line from the random point and parallel to the x-axis.
- (6) Count the number of intersections of this line with the boundary
- (7) Test whether the random point falls within the boundary according to Rule 1 (an odd or even number of intersections).
- (8) If the above test succeeds, increase the counter of accepted points by one.
- (9) Repeat the processes of 5, 6, 7 and 8 until the number of points assigned is reached.
- (10) Compute the area by dividing the accepted points by the total number of random points, and multiplying this ratio by the enclosing rectangular area.

The above procedure is simulated as a flow chart shown in Figure 1.

iii. Selecting Boundary Segments

The boundaries of a watershed can be defined by the given coordinates of successive points along the boundary (in a clockwise direction) and considering the boundary between each pair of successive points to be a straight-linear segment. The actual boundaries could be approximated as closely as desired by increasing the number of such segments; but, the user should note that the more segments chosen the more computing time and user's time are required. A later example will show the effects of many irregularities of natural boundaries such as lake shorelines and the watershed may be averaged by reducing the boundary planform to a simplified polygon. The general principle of this procedure is to represent the planform

Figure 1: Flowchart for Solving Integrated Area



with as few sides as possible without changing the basic shape of the boundary.

iv. Convergence of Weights

Because the Monte Carlo method relies on the laws of probability, a large number of random trials should be taken in the interest of precision. The method normally used to estimate the relationship of sample size and accuracy is the large-sample normal approximation. Using the Central-Limit Theorem, the binomial distribution can be approximated by a normal distribution for a large N , where N is the total number of trials. The sampling error of any statistic is proportional to $1/(N)^{1/2}$. The convergence of the Thiessen weights is a statistical convergence i.e., the probable error of estimation is proportional to $1/(N)^{1/2}$. A detailed discussion was given by Shih and Hamrick (1974).

WEIGHTED SUB-AREAS

The average amounts of environmental elements such as water quality, ecology system, and land use, etc. over a specific area is required in many water resource problems. Thus, mean value problems can be solved by sampling techniques. For convenience let variables A_1, \dots, A_k , and X_1, \dots, X_k be the subareas and measured values of stations 1, ...k, respectively. Then the estimated weighted average amounts for the region are

$$X = W_1X_1 + \dots + W_kX_k = \sum_{i=1}^k W_iX_i \quad (7)$$

where $W_i = A_i/A$, weighted area; and

$A = A_1 + \dots + A_k$, total area.

In most cases the values of X_i are obtained first by laboratory experiments or field measurements, and then the values of W_i are estimated based on the

following four methods:

I. Simple Arithmetical Method: The values of A_i are assumed equal to one.

Equation 6 can be rewritten as

$$\bar{X} = \sum_{i=1}^k X_i/k \quad (8)$$

Equation 8 is the simplest method which can give a good estimation of average value in a flat area under the condition of measurements that are uniformly distributed and the individual measurement does not vary widely from the true average. However, this method does not take into account the measurements outside, but near the boundaries of the area.

- II. Thiessen Polygon Method: Thiessen (1911) developed a method which attempts to allow for nonuniform distribution of measurements by providing a weighting factor for each measurement. The measured points are plotted on a map, and connecting lines are drawn. Perpendicular bisectors of these connecting lines form polygons around each measured point. The sides of each polygon are the boundaries of the effective area assumed for the measured point. The values of A_i are determined by the methods indicated in the section of bounded area computation. However, the limitation of this method is its inflexibility; for instance, a new polygon being required every time there is a change in sampling location. Also, the method makes no attempt of overcome the orographic influences.
- III. Stratified Method: This method is a plan by which the region is divided into homogeneous subregions or strata. In computing a strata the analyst can make full use of his knowledge of orographic effects or other influential factors. After constructing the strata, the following equation is used to calculate the mean values, \bar{X} ,

$$\bar{X} = \sum_{i=1}^M \sum_{j=1}^{k_i} W_{ij} X_{ij} \quad (9)$$

where

M = total number of strata

k_i = total number of observed stations in the i th stratum

i, j = index of stratum and observed station, respectively; and

W_{ij} = observed value of the j th station in the i th stratum

The values of A_{ij} can be determined by either simple arithmetical method or the Thiessen method. If a simple arithmetical method is used, then equation 9 is reduced as

$$\bar{X} = \sum_{i=1}^M W_i \bar{X}_i \quad (10)$$

where

$W_i = A_i/A$, weighted area of the i th stratum; and

$\bar{X}_i = \sum_{j=1}^{k_i} X_{ij}/k_i$, average observed area of i th stratum.

If a Thiessen method is used to perform the subpolygon for each station X_{ij} , then equation 9 is used to calculate the weighted area. The greatest limitation of the stratified method is also its inflexibility.

IV. Modified Monte Carlo Method: As indicated in previous sections, the Thiessen polygon and stratified methods suffer from their inflexibility in that a new Thiessen diagram is required every time there is a change in the sampling location or a recorded station with missing data. This limitation can be overcome by the modified Monte Carlo method (Shih and Hamrick, 1975).

a. Procedures of Computation

The following procedures are used to compute an area weighted to each measured station:

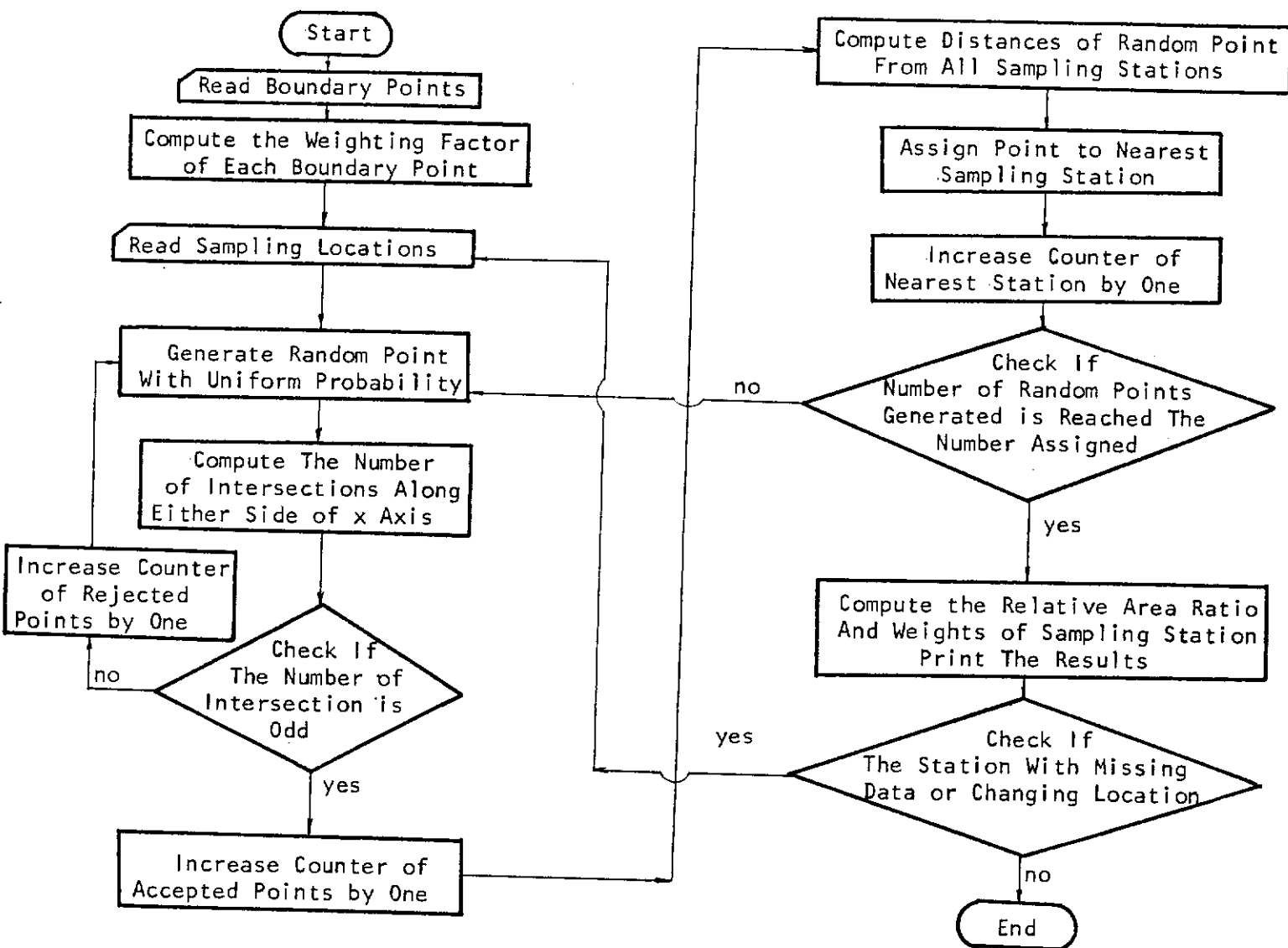
1. Compute the weighting factor of each boundary node which relies on the procedures 1, 2 and 3 as indicated in the previous section of the Monte Carlo method.
2. Determine whether a random point falls within any shape boundary which is based on the procedures 4, 5, 6, 7 and 8 as indicated in the previous section of Monte Carlo method.
3. Assign the random point which is falling within the boundary to the nearest measuring point.
4. Repeat processes 2 and 3 until a predetermined large number of points are reached.
5. Compute the relative area ratio of the bounded region to the enclosing rectangle by dividing the number of accepted points by the total number of random points.
6. Calculate the computed weights of each measuring point by dividing the number of points assigned to each measuring point by the total number of accepted points.
7. Check whether the sampling location is changing.
8. If the response to 7 is yes, the processes from 2 to 7 are repeated.

The above procedures are simulated in a flow chart in Figure 2.

b. An Irregular or L-Shaped Watershed

An expected accuracy of computation by this Monte Carlo method depends upon not only the number of random points, but also the shape of the watershed. The number of random points that affects the sample error has been discussed in a previous section. However, the

Figure 2: Flowchart for Computing the Weighted Sub-area.



efficiency of this Monte Carlo method affected by the shape formed by the watershed should also be discussed because a large number of random points fall off the outside boundaries of the watershed which differs greatly from that of the enclosing rectangle. This difficulty can be overcome by following three techniques:

- i. Equal Rectangles: A watershed is enclosed by a number of smaller rectangles of equal area that have a common edge which cuts the watershed. In order for this method to be used more widely, the following relationship should be introduced. Let A_1, \dots, A_n be the relative area ratio falling within the boundaries of sub-watershed 1, ..., n. Then the new relative area ratio, R_1, \dots, R_n , of sub-watershed 1, ..., n are equal to

$$\begin{aligned} R_1 &= A_1 / \sum_{i=1}^n A_i \\ &\vdots \\ R_n &= A_n / \sum_{i=1}^n A_i \end{aligned} \quad (11)$$

The final computed weights, W_1, \dots, W_m , of rainfall stations 1, ..., m are equal to

$$\begin{aligned} W_1 &= \sum_{i=1}^n R_i E_{i1} \\ &\vdots \\ W_m &= \sum_{i=1}^n R_i E_{im} \end{aligned} \quad (12)$$

where E_{ij} is the computed weight of rainfall station j in subrectangle i ; j includes the rainfall station from 1 to m and i includes the subrectangle from 1 to n . For example,

m is the total number of rainfall stations and n is the total number of subrectangles.

- ii. Unequal Rectangles: The technique of unequal rectangles is similar to the equal rectangles method except that the watershed is enclosed by a number of smaller unequal rectangles. Let S_1, \dots, S_n represent the area of enclosing rectangles 1, ..., n; A_1, \dots, A_n and R_1, \dots, R_n are defined in the case of equal rectangles. The value of R_i is

$$\begin{aligned} R_1 &= A_1 S_1 / \sum_{i=1}^n A_i S_i \\ &\vdots \\ R_n &= A_n S_n / \sum_{i=1}^n A_i S_i \end{aligned} \quad (13)$$

The final computed weights, W_1, \dots, W_m of rainfall stations, 1, ..., m, are similar to equation 12, except that the R_i values are replaced by equation 13.

- iii. Single Rectangle: The more random points chosen, the greater the accuracy of the estimates obtained. Therefore, in a watershed which has a lower relative area ratio, the single-rectangle technique is still applicable by increasing the random trials. A detailed technique of application was given by Shih and Hamrick (1975).

c. New Thiessen Coefficients for Missing Data

As Linsley et al. (1958) indicated the greatest limitation of the Thiessen method is its inflexibility, because a new Thiessen polygon is required every time there is a change in the gage network. This modified Monte Carlo method can be used to overcome this limitation.

In general, there are two cases of missing data. Case 1: The missing data of each rainfall station are priorly known, and any missing periods of record are assigned as a new station set. The distance of a random point from all rain measuring stations is calculated simultaneously in each station set, and the random point is assigned to the nearest rain measuring station in each set. Case 2: The missing data of each rainfall station is posteriorly known, i.e., how many stations with missing data are unknown. In this case, if a station with a missing record is found, then that station is omitted and a new gage network is considered. Based on this new gage network, a computed weight is performed by a repeating procedure. A detailed description of these procedures will be discussed in the section of computer program.

COMPUTER PROGRAM

Based on Equation 1 of finite segments, procedures for computation in Monte Carlo and modified Monte Carlo methods, a systematic flow chart for the computer program development is shown in Appendix 1. Nomenclatures for the computer program are listed in Appendix 2. The systematic flow chart was converted to a computer program called E084 for the CDC 3100 computer with Fortran IV language. The users manual for the E084 program is also presented in Appendix 4. An example of using the Upper Kissimmee Basin for input is demonstrated on Appendix 5. The results of computer output are also shown in Appendix 6.

EXAMPLES OF APPLICATION

Four examples are used to illustrate the application of computing the bounded and weighted sub-areas.

EXAMPLE 1. SIMPLE GEOMETRICAL SHAPE

In order to test the applicability of these newly developed techniques, an irregular area as shown in Figure 3 not only has inactive areas within the figure, but also can be calculated easily by the triangular rule. As mentioned in the previous sections, the direction used in setting boundary nodes is a very important feature in some methods, because an improper procedure can cause a serious error. In order to investigate this nature, as Figure 3 illustrates, four different sequences used to select the coordinates are demonstrated:

Case 1A: 1-2-3-4-5-6-7-8-9-10-6-11-12-13-14-11-5-1,

Case 2A: 1-2-3-4-5-6-10-9-8-7-6-11-12-13-14-11-5-1,

Case 1B: 1-2-3-4-5-6-7-8-9-10-11-12-13-10-9-14-6-5-1,

Case 2B: 1-2-3-4-5-6-14-9-10-13-12-11-10-9-8-7-6-5-1.

Based on these four cases, several methods are used to obtain the areas. The results are shown in Table 1. As can be seen from Table 1, the cases of 1A and 1B have a serious error by using the methods of finite segments and digitizer. The results of areas calculated by the Monte Carlo method with 2000 random walks are all the same values in four cases. This implies that the technique used to select the consecutive points along the boundary in either a clockwise or counterclockwise direction, or a combination of both directions, gives no difference in the results obtained by using the Monte Carlo method. This is a very useful tool when the directions used to select the boundary nodes are mixed up in both directions. The accuracy of Simpson's method is directly proportional to the number of divisions. For example, the number of eight and ten offsets as shown in Figure 3 are called Cases 1 and 2, respectively. The calculating results are also shown in Table 1. As Table 1 shows, Case 2 has a better solution than Case 1. The results obtained by the coordinates and weighed method give a good agreement with the results obtained by the triangular rule.

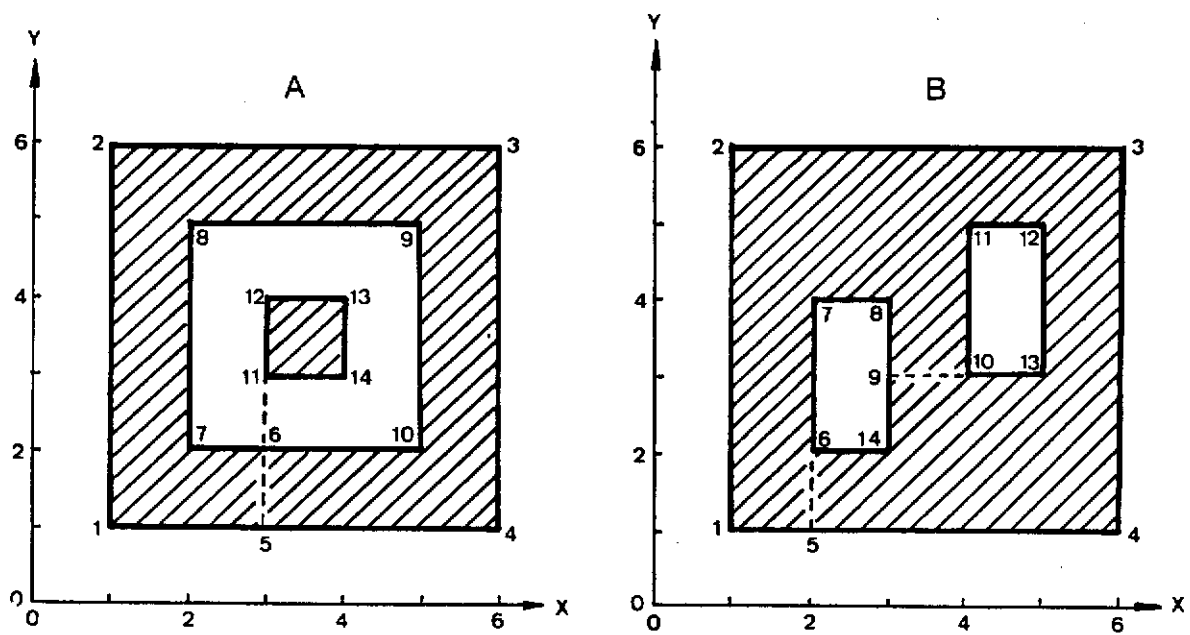


Fig. 3. Area Under an Irregularly Shaped Form.

Table 1. Comparison of the results of bounded area obtained by Monte Carlo method with other methods.

Diff. Cases	Graphical Method			Arithmetical Method		Computerized Method	
	Coordinate Squares	Digitizer Method	Weighed Method	Triangular Rule	Simp. Rule	Monte Carlo	Finite Segments
1A	16.97	34.03	17.18	17	18.3	17.06	35
2A	16.97	17.03	17.18	17	17	17.06	17
1B	20.84	29.82	19.98	21	20	20.81	29
2B	20.84	20.92	19.98	21	21	20.81	21

EXAMPLE 2. KISSIMMEE RIVER BASIN

The Kissimmee River basin as shown in Figure 4, with sixteen rainfall stations is used as a practical problem to exemplify the technique of a single rectangle as mentioned in the previous section of modified Monte Carlo method. It should be noted that the coordinates chosen for enclosing the rectangle are an important factor. The coordinates should be chosen to make the enclosing rectangle as small as possible so that as high an area ratio as possible can be obtained. The results with random trials of 2,000, 4,000 and 6,000 are shown in Table 2. Comparison of the single rectangle results with the actual values as shown in Table 2, indicates that the single rectangle is applicable to a watershed with a long and narrow shape.

EXAMPLE 3. UPPER KISSIMMEE RIVER BASIN

The Upper Kissimmee Basin, as shown in Figure 5 with two different cases of selecting boundary points such as 103, demonstrates the planform of watershed boundary affecting the accuracy and computing time of estimation. The results showed that the computing time was reduced about 70% in the 10 points case and accuracy did not change appreciably. Therefore, a case with boundary points

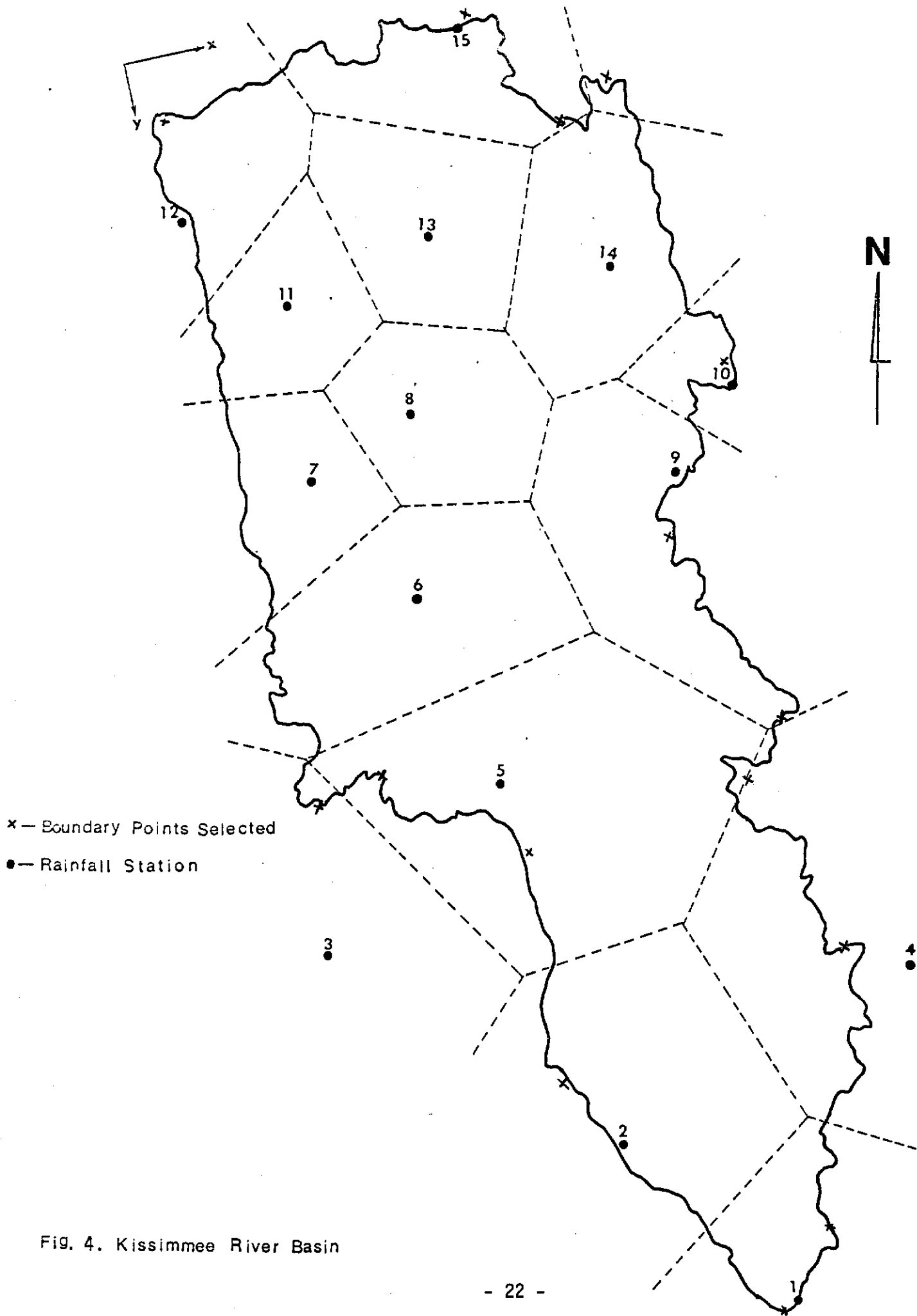


FIG. 4. Kissimmee River Basin

Table 2. Computed weights and relative area ratio of Figure 4 shown.

Rainfall station	Graphical method	Monte Carlo method		
		No. of random points		
		2000	4000	6000
1	0.029	0.036	0.034	0.030
2	0.102	0.107	0.105	0.107
3	0.002	0.0	0.0	0.001
4	0.058	0.054	0.057	0.057
5	0.166	0.179	0.188	0.190
6	0.113	0.117	0.118	0.114
7	0.053	0.044	0.042	0.044
8	0.071	0.054	0.069	0.068
9	0.089	0.081	0.079	0.084
10	0.015	0.016	0.013	0.012
11	0.053	0.047	0.043	0.046
12	0.043	0.044	0.042	0.046
13	0.080	0.096	0.085	0.080
14	0.078	0.066	0.073	0.071
15	0.046	0.058	0.051	0.050
16	0.002	0.002	0.001	0.001
Ratio of area		0.621	0.624	0.630

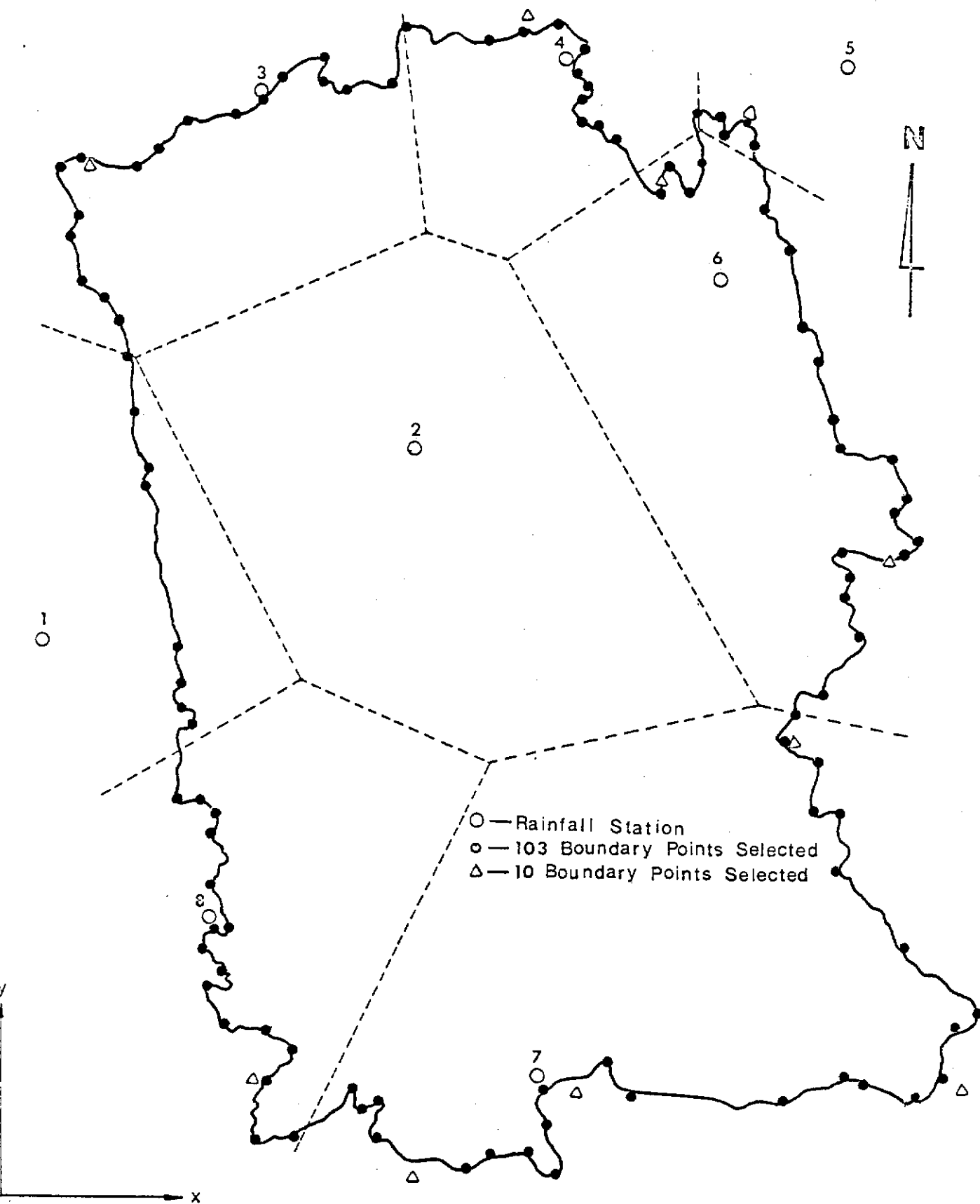


Fig. 5. Upper Kissimmee River Basin

is also used to demonstrate the bounded area and Thiessen coefficients computations. Comparison of bounded area obtained by different random points are given in Table 3, showing that the bounded area can improve the accuracy by increasing the random points. However, too many random points are not recommended in practical application because a longer computing time is involved for this little improvement in accuracy. In general, 2,000 - 6,000 random points can give quite good results. As can be seen from Table 3, the area computed by finite segment method is 1605 square miles which is very close to the area of 1607 square miles given by U.S.D.A. (1974).

Comparisons of the results of Thiessen coefficients affected by different random points are shown in Table 4. As can be seen from Table 4, 2,000 - 6,000 random points also can give quite satisfactory results from Thiessen coefficients. A detailed discussion on the relationship between random points and accuracy of Thiessen coefficients computation was given by Shih and Hamrick (1975).

Table 3. Comparison of the results of bounded area obtained by the different random points.

Bounded Area	Finite Segments Method	MONTE CARLO METHOD						
		Number of Random Points Assigned						
		500	1000	2000	4000	6000	10000	20000
Sq. Miles	1605	1693	1712	1648	1628	1617	1612	1600
Percent of Deviation	0.0	5.50	6.70	2.68	1.43	0.75	0.44	0.31

Table 4. Comparison of the results of Thiessen Coefficients affected by different random points.

Rainfall Stations	Thiessen Polygon	MODIFIED MONTE CARLO METHOD						
		Number of Random Points Assigned						
		500	1000	2000	4000	6000	10000	20000
Lake Alfred	0.034	0.029	0.023	0.029	0.034	0.034	0.037	0.034
Kissimmee II	0.288	0.274	0.279	0.296	0.296	0.289	0.295	0.296
Isleworth	0.091	0.055	0.087	0.088	0.083	0.088	0.090	0.087
Orlando	0.052	0.068	0.063	0.054	0.052	0.049	0.051	0.051
Bithlo	0.001	0.003	0.002	0.003	0.002	0.001	0.001	0.001
Lake Hart	0.160	0.140	0.150	0.147	0.152	0.153	0.150	0.156
Ind. Lake Estates	0.259	0.309	0.275	0.263	0.269	0.267	0.263	0.263
Mount. Lake Estates	0.115	0.121	0.122	0.120	0.113	0.118	0.112	0.118

This example was also used to test the applicability of the missing data case. The new computed Thiessen weights with 2,000 random points for missing data of each station in these eight stations are shown in Table 5. The results indicate that the modified Monte Carlo method technique is capable of overcoming difficulties due to changes in network.

EXAMPLE 4. ESTIMATING THE CONCENTRATION OF NUTRIENTS IN CONSERVATION AREA 2A.

Conservation Area 2A, as shown in Figure 6, with 15 measuring stations is used to exemplify the application of the Modified Monte Carlo technique. It should be noted that the coordinates chosen for enclosing the rectangle are an important factor. The coordinates should be chosen to make the enclosing rectangle as small as possible so that as high an area ratio as possible be obtained. The results of the weighting area of 15 measured stations, obtained by both methods of the Thiessen polygon and Monte Carlo technique, are shown in

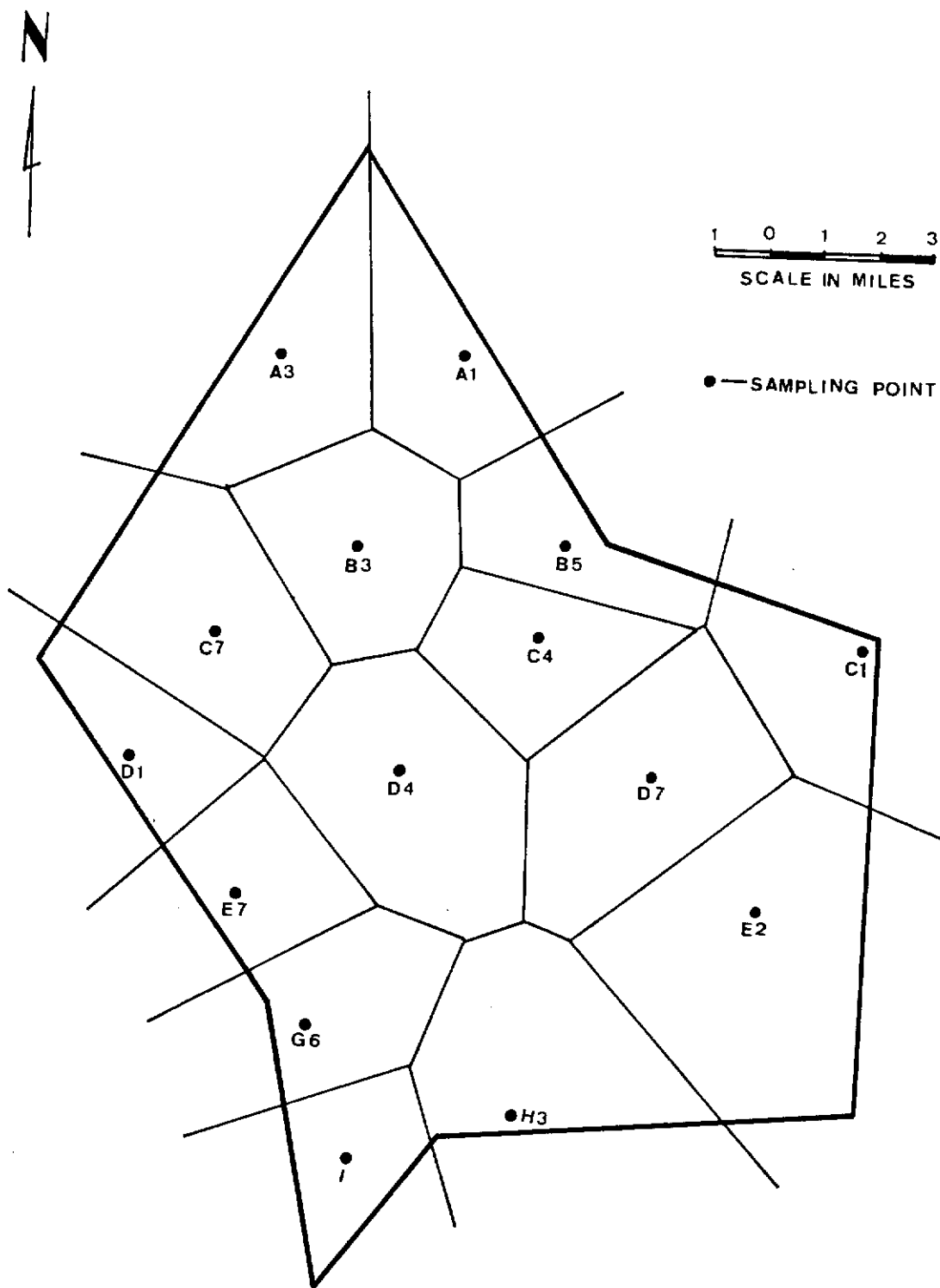


Fig. 6.. Sampling Locations of Conservation Area 2A.

Table 5. Computed new weights and relative area ratio in missing data for the upper Kissimmee River Basin as shown in Figure 5.

Rainfall station	Station with missing data							
	0	1	2	3	4	5	6	7
Lake Alfred	0.034	-	0.109	0.034	0.034	0.034	0.034	0.034
Kiss. II	0.300	0.324	-	0.348	0.300	0.300	0.386	0.341
Isleworth	0.090	0.090	0.134	-	0.127	0.090	0.090	0.090
Orlando	0.059	0.059	0.063	0.100	-	0.059	0.088	0.059
Bithlo	0.001	0.001	0.001	0.001	0.001	-	0.040	0.001
Lake Hart	0.155	0.155	0.271	0.155	0.177	0.156	-	0.156
Indian Lake Estates	0.258	0.258	0.283	0.258	0.258	0.258	0.258	-
Mountain Lake Estates	0.103	0.114	0.140	0.103	0.103	0.103	0.103	0.320
Ratio of area	0.580	0.580	0.580	0.580	0.580	0.580	0.580	0.580

in Table 6. Comparisons of the Monte Carlo results with the values obtained by the graphical method indicates that the Monte Carlo method is applicable to a region for obtaining an area weighted to each measure station. The sampling program of Chloride and Silica concentrations in Conservation Area 2A was conducted by Gleason (1974). The average of the concentrations are computed by using the simple arithmetical method, Thiessen polygon and Monte Carlo method. The results are also shown in Table 6. As Table 2 shows, the averages of Chloride and Silica concentrations obtained by the Thiessen polygon and Monte Carlo methods give a similar result.

In order to investigate the applicability of the modified Monte Carlo method to the water quality sample program, the average of 49 sample data and percentage of deviation among each method were listed on Table 7. Some abbreviation terms are used in Table 7. For example, the simple arithmetic method with 49 sample data is called SMM49 and with 15 samples is called SMM15; the Thiessen coefficient polygon and Monte Carlo method with 15 sample data are called TCP15 and MCM15, respectively. As Table 7 shows, the following results can be drawn:

- (1) Based on the statistical viewpoint, the more samples taken the more accurate should be the estimation of the mean value. In other words, the sample data with 49 points should be better than the 15 points, i.e., the average values of 113.96 ppm of chloride and 17.20 ppm of Silica have more reliable estimation than others. Therefore, the results of SMM49 are better than the SMM15.
- (2) When the sample sizes are as few as 15 points, the estimations based on TCP15 and MCM15 are better than SMM15 because the deviation percentages between SMM49 and SMM15 are about 17% in Cl and 11% in SiO_2 , but between SMM49 and TCP15 are only 7%,

Table 6. Comparison of the results of mean chloride and silica concentrations obtained by modified Monte Carlo method with graphical method.

Sampling Stations	Weighted Sub-area		Chloride Concentration, ppm			Silica Concentration, ppm		
	Thiessen ^a Polygon	Monte Carlo Method	Observed Data	Thiessen ^a Polygon	Monte Carlo Method	Observed Data	Thiessen ^a Polygon	Monte Carlo Method
A ₁	0.0529	0.0524	243	12.86	12.72	33.2	1.76	1.74
A ₃	0.0673	0.0598	44	2.96	2.63	0.5	0.03	0.03
B ₃	0.0712	0.0806	112	7.97	9.02	10.2	0.73	0.82
B ₅	0.0431	0.0444	179	7.71	7.94	23.9	1.03	1.06
C ₁	0.0484	0.0412	166	8.04	6.84	21.6	1.05	0.89
C ₄	0.0543	0.0585	182	9.88	10.65	23.7	1.29	1.39
C ₇	0.0819	0.0718	112	9.17	8.04	12.2	1.00	0.88
D ₁	0.0371	0.0339	152	5.64	5.15	17.9	0.66	0.61
D ₄	0.0991	0.1157	114	11.30	13.19	21.6	2.14	2.50
D ₇	0.0932	0.1004	115	10.71	11.55	21.6	2.01	2.17
E ₂	0.1381	0.1219	81	11.19	9.87	18.7	2.58	2.28
E ₇	0.0442	0.0451	152	6.71	6.86	19.2	0.85	0.87
G ₆	0.0514	0.0527	137	7.04	7.23	22.5	1.16	1.19
H ₃	0.0818	0.0888	82	6.71	7.29	16.9	1.38	1.50
I	0.0360	0.0328	134	4.82	4.40	21.6	0.78	0.71
Total	1.0000	1.0000	2005			285.3		
Mean Value			133.67*	122.71	123.28	19.02	18.45	18.64

^a The areas of Thiessen polygon are obtained by graphical method.

* The mean values are obtained by arithmetical method.

and between SMM49 and MCM15 are also only 8%.

- (3) Based on the methods of TCP15 and MCM15, the error of estimation average can be reduced at least 53% in Cl and 21% in SiO₂. This implies that either the TCP15 or the MCM15 can give a better result of the average value over a specific area.
- (4) The results of modified Monte Carlo method and Theissen coefficient polygon are similar. This implied that the Modified Monte Carlo method not only can be used to estimate the mean value over a specific area, but also can be computed directly by a computerized procedure.

Table 7. Percentage differences and average concentrations of chloride and silica in Conservation Area 2A.

Methods	Cl deviation %			Average Concentration ppm	SiO ₂ deviation %			Average Concentration ppm
	SMM49	SMM15	TCP15		SMM49	SMM15	TCP15	
SMM49*				113.96				17.20
SMM15	17.30			133.67	10.58			19.02
TCP15	7.68	8.93		122.71	7.27	3.09		8.45
MCM15	8.18	8.43	0.46	123.28	8.37	2.04	1.03	18.64

- * SMM49 is the simple arithmetic method with 49 sample data.
 SMM15 is the simple arithmetic method with 15 sample data.
 TCP15 is the Theissen coefficients with 15 sample data.
 MCM15 is the Monte Carlo method with 15 sample data.

RESULTS AND DISCUSSION

After applying the techniques to several practical problems, the following conclusions were drawn:

1. The accuracy of the graphical method is highly dependent upon the skill of the analyst and the correctness of the instruments. Although the results obtained by the coordinate squares, planimeter and weighing technique are in good agreement with the results obtained by the triangular rule, some observations must be mentioned. For example, the nonuniform thickness of the paper and varying humidity can significantly affect the accuracy of weighing measurement. Poor setting of the planimeter scale bar, and failure to check for the scale constant by tracing a known area, can cause an error of planimeter measurement. Using coordinates which are too large make it difficult to estimate the partial blocks from which an error of computation can occur. The cases of 1A and 1B will have a serious error if the digitizer is used. These results show that the digitizer is highly dependent upon the direction used to digitize the boundary coordinates.
2. The accuracy of the arithmetical technique is dependent upon the number of offsets used to divide the entire region. Case 2 with ten offsets has a better solution than Case 1 with only eight offsets. If a region has a very irregular shape and large size, then this arithmetical method is very tedious and cumbersome to use.
3. The signs of latitudes, departures, and starting points are easily confused in the methods of DMD and coordinates. Although finite

segments can overcome some limitations such as a figure having some inactive areas within the figure, as Case 1A and 1B shows; it is strongly dependent upon the direction used to select the boundary nodes but also applicable to any complicated form of figures with arbitrary sign of latitudes, departures and starting points.

4. The modified Monte Carlo method has been successfully applied to compute the weighted sub-area. The result also indicated that only the modified Monte Carlo method can be simulated directly by a computerized procedure to compute the weighted sub-area to each measured station.
5. The modified Monte Carlo method can be applied to compute the weighted sub-area not only for rainfall stations but also for any type of estimating mean value over a specific area. For example, a mean chloride and silica concentration in Conservation Area 2A can be computed based on the modified Monte Carlo method.
6. The modified Monte Carlo method has been extended to compute the new weighting factors when data are missing.

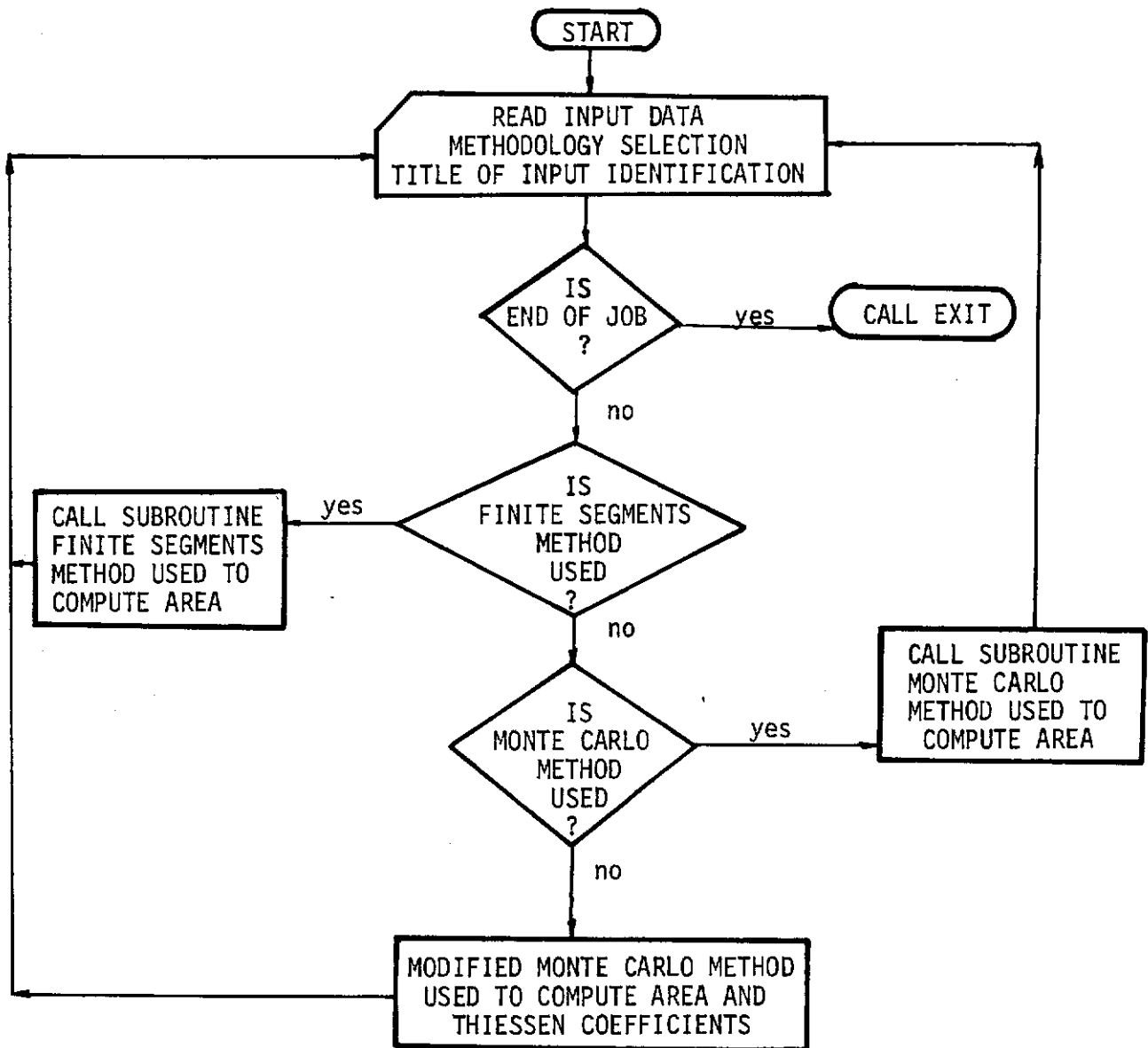
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APPENDICES

Appendix 1. Systematic Flow Chart For Computer Program Development



Appendix 2. Nomenclature For Computer Programs.

Variables of Input:

- IN - Methodology indicator, the finite segments method used
 - 1 = Finite Segments Method Used
 - 2 = Monte Carlo Method Used
 - 3 = Modified Monte Carlo Method Used
 - 0 = End of Job
- TITLE - Input data identification
- N - The number of boundary points chosen
- M - The number of measuring stations
- NSET - The number of Monte Carlo Points assigned
- XMIN - Minimum range in X axis
- XMAX - Maximum range in X axis
- YMIN - Minimum range in Y axis
- YMAX - Maximum range in Y axis
- FAC - The scale of feet used in per unit of length, zero means the dimensionless of area
- X - X coordinate of the boundary point
- Y - Y coordinate of the boundary point
- AX - X coordinate of the measuring station
- AY - Y coordinate of the measuring station

Variables of Output:

- AF - Relative area ratio
- AREA - Total area in acres
- I - Measuring station identification
- WF(I) - Thiessen coefficient of station I
- FACT - Area converting factor in unit of acres
- SUM - Total area in acres computed by finite segments method.

Appendix 3. Users Manual For E084

For Program to Compute the Areas and Thiessen
Coefficients Based on Finite Segments, Monte
Carlo and Modified Monte Carlo Methods

E084
(FCD AC)

Program Limitations

1. Limit of 400 boundary points per area
2. Limit of 500 measuring station per area

Requisition for Computer Work

Estimated Time - 5 seconds is needed in Finite Segment per area,
15 seconds is needed in Monte Carlo method per 1000 random points
20 seconds is needed in Modified Monte Carlo Method per
1000 random points

Category - Production run

Job Run No. - E084

Disk - 6000

FORMAT INFORMATION

Symbols used to indicate the proper method for numbers or letters entered
in card columns shown are :

RJ - indicates that a whole integer number must be right justified
in card columns shown

DP - indicates that the number must have a decimal point indicated in
one of the card columns.

A - any alpha-numeric character.

CARD FORMAT INFORMATION

First Card: Control and Parameters Card

<u>C.C.</u>	<u>Symbol</u>	<u>Description</u>
1-5	RJ	An integral value is required. 1, 2, and 3 indicate that the finite segments, Monte Carlo, and Modified Monte Carlo are used, respectively.
11-80	A	Title Input data identification.

Second Card: Parameters Card

<u>C.C.</u>	<u>Symbol</u>	<u>Description</u>
Type I: when the integer 1 is shown in first card on column 5		
1-10	RJ	Total number of boundary points
11-20	DP	Scale of feet used in per unit of length
Type II: When the integer 2 is shown in first card on column 5		
1-10	RJ	Total number of random points assigned
11-20	RJ	Total number of random points assigned
21-30	DP	Minimum value of enclosing rectangle coordinates along X-axis
31-40	DP	Maximum value of enclosing rectangle coordinates along X-axis
41-50	DP	Minimum value of enclosing rectangular coordinate along Y-axis
51-60	DP	Maximum value of enclosing rectangular coordinate along Y-axis
61-70	DP	Scale 4 feet used in per unit of length
Type III: When the integer 3 is shown in first card on column 5		
1-10	RJ	Total number of boundary points
11-20	RJ	Total number of rain measuring stations. If a relative area ratio of study area to enclosing rectangle is expected, only the value of 1 should be used.
21-30	RJ	Total number of random points assigned.
31-40	DP	Minimum value of enclosing rectangular coordinate along X-axis
41-50	DP	Maximum value of enclosing rectangular coordinate along X-axis
51-60	DP	Minimum value of enclosing rectangular coordinate along Y-axis
61-70	DP	Maximum value of enclosing rectangular coordinate along Y-axis
71-80	DP	Scale of feet used in per unit of length

Third Card: Boundary Points Coordinates Card(s)

<u>C.C.</u>	<u>Symbol</u>	<u>Description</u>
1-5	DP	
11-15	DP	
31-35	DP	
41-45	DP	x coordinate value of boundary segments choosing in clockwise direction
51-55	DP	
61-65	DP	
71-75	DP	
6-10	DP	
16-20	DP	
26-30	DP	
36-40	DP	
46-50	DP	y coordinate value of boundary segments choosing in clockwise direction.
56-60	DP	
66-70	DP	
76-80	DP	

Note: The maximum boundary points included per card is only 8 points. Therefore, the card can be used as much as required in number of stations. For example, 7 cards are needed in 50 boundary points.

Measuring Stations Coordinate Card(s): This card is required only when the integer 3 is shown in the first card on column 5.

<u>C.C.</u>	<u>Symbol</u>	<u>Description</u>
1-5	DP	
11-15	DP	
21-25	DP	
31-35	DP	
41-45	DP	x coordinate of rain measuring station
51-55	DP	
61-65	DP	
71-75	DP	
6-10	DP	
16-20	DP	
26-30	DP	
36-40	DP	
46-50	DP	y coordinate of rainfall measuring station
56-60	DP	
66-70	DP	
76-80	DP	

Note 1: The maximum measuring stations involved per card is only 8 stations. Therefore, use as many new cards as necessary.

Note 2: Use as many new control and title cards with succeeding cards as necessary. The last card must be present as a blank form.

Appendix 4. Example of Upper Kissimmee River Basin Input


```

1  FINITE SEGMENTS METHOD USED TO COMPUTE THE AREA
   10 31000.0
3.30 0.18 2.00 0.98 0.50 8.65 4.10 9.90 5.20 8.50 5.85 9.15 7.05 5.35 6.25 3.75
7.95 0.92 4.60 0.90
2  MONTE CARLO METHOD USED TO COMPUTE THE AREA
   10 10000 0.000 8.000 0.000 10.000 31000.0
3.30 0.18 2.00 0.98 0.50 8.65 4.10 9.90 5.20 8.50 5.85 9.15 7.05 5.35 6.25 3.75
7.95 0.92 4.60 0.90
3  MODIFIED MONTE CARLO METHOD USED TO COMPUTE THIESSEN COEF.
   10 10000 0.000 8.000 0.000 10.000 31000.0
3.30 0.18 2.00 0.98 0.50 8.65 4.10 9.90 5.20 8.50 5.85 9.15 7.05 5.35 6.25 3.75
7.95 0.92 4.60 0.90
3.20 6.25 1.90 9.30 4.40 9.60 6.75 9.50 5.70 7.70 4.25 .90 1.55 2.30

```

Appendix 5. Example of Upper Kissimmee River Basin Output

FINITE SEGEMENTS METHOD USED TO COMPUTE THE AREA

NUMBER OF BOUNDARY SEGMENTS = 10
AREA CONVERTING FACTOR = 22061.52433

TOTAL AREA IN ACRES = 1026936.380829

MONTE CARLO METHOD USED TO COMPUTE THE AREA

NUMBER OF BOUNDARY SEGMENTS = 10

NUMBER OF RANDOM POINTS = 10000

X-AXIS MINIMUM = 0

X-AXIS MAXIMUM = 8.0000

Y-AXIS MINIMUM = 0

Y-AXIS MAXIMUM = 1.0000

AREA CONVERTING FACTOR = 22861.52433

RELATIVE AREA RATIO = .584700 TOTAL AREA IN ACRES = 1031949.862244

MODIFIED MONTE CARLO METHOD USED TO COMPUTE THIESSEN COEF.

NUMBER OF BOUNDARY SEGMENTS = 10
NUMBER OF RAINFALL STATIONS = 7
NUMBER OF RANDOM POINTS = 10000
X-AXIS MINIMUM = 0
X-AXIS MAXIMUM = 8.0000
Y-AXIS MINIMUM = 0
Y-AXIS MAXIMUM = 13.0000
AREA CONVERTING FACTOR = 22061.52433

RELATIVE AREA RATIO = .584700 TOTAL AREA IN ACRES = 1031949.862244

COMPUTED WEIGHT OF RAINFALL STATION	1	=	.322388
COMPUTED WEIGHT OF RAINFALL STATION	2	=	.090474
COMPUTED WEIGHT OF RAINFALL STATION	3	=	.050966
COMPUTED WEIGHT OF RAINFALL STATION	4	=	.000855
COMPUTED WEIGHT OF RAINFALL STATION	5	=	.149991
COMPUTED WEIGHT OF RAINFALL STATION	6	=	.262699
COMPUTED WEIGHT OF RAINFALL STATION	7	=	.122627

Appendix 6. E084 - Computer Program
(FCDAC)

```

      PROGRAM FCDAC
C   X IS THE X COORDINATE OF THE BOUNDARY
C   Y IS THE Y COORDINATE OF THE BOUNDARY
C   NN IS A WEIGHTING FACTOR FOR NODE POINT OF BOUNDARY
C   IA AND IX ARE INITIAL ODD INTEGER FOR USING IN RANDU SUBROUTINE
C   N IS THE NUMBER OF BOUNDARY POINTS CHOSEN
C   M IS THE NUMBER OF RAIN MEASURING STATION
C   NSET IS THE NUMBER OF RANDOM POINTS EXPECTED
C   XMIN AND XMAX ARE THE MINIMUM AND MAXIMUM RANGE IN X AXIS
C   YMIN AND YMAX ARE THE MINIMUM AND MAXIMUM RANGE IN Y AXIS
C   AX IS THE X COORDINATE OF THE RAIN MEASURING STATION
C   AY IS THE Y COORDINATE OF THE RAIN MEASURING STATION
C   FAC IS THE FEET USED IN PER UNIT ZERO MEANS DIMENSIONLESS OF AREA
      REAL L(500)
      DIMENSION X(400),Y(400),AX(500),AY(500),WF(500),NS(500),YY(400),
     1 NN(400),TITLE(9)
      NR=60
      NW=61
C   READ INPUT DATA
83 READ(NR,3) IW,TITLE
      3 FORMAT(15,3X,9A8)
      IA=5
      IX=7
C   CHECK WHETHER THE END OF STATION SET
      IF(IW.EQ.0) GO TO 85
      WRITE(NW,5) TITLE
      5 FORMAT(1H1,/,5X,9A8//)
      IF(IW.EQ.1) GO TO 109
      IF(IW.EQ.2) GO TO 110
      READ(NR,8) N,M,NSET,XMIN,XMAX,YMIN,YMAX,FAC
      8 FORMAT(3I10,5F10,3)
      IFAC=FAC+0.01
      FACT=FAC*FAC/43561.2
      IF(IFAC.EQ.0) FACT=1.0
      WRITE(NW,108) N,M,NSET,XMIN,XMAX,YMIN,YMAX,FACT
108 FORMAT(/,10X,29HNUMBER OF BOUNDARY SEGMENTS =,I4/10X,29HNUMBER OF
1RAINFAILL STATIONS =,I4/1 X,25HNUMBER OF RANDOM POINTS =,I6/10X,16H
2X-AXIS MINIMUM =,F8.4/1 X,16HX-AXIS MAXIMUM =,F8.4/10X,16HY-AXIS M
3INIMUM =,F8.4/10X,16HY-AXIS MAXIMUM =,F8.4/10X,24HAREA CONVERTING
4FACTOR =,F15.5//)
      READ(NR,9) (X(I),Y(I),I=1,N)
      9 FORMAT(16F5,1)
      READ(NR,4) (AX(I),AY(I),I=1,M)
      4 FORMAT(16F5,1)
C   ADD A SMALL VALUE TO EACH NODE FOR ORTAING THE NN FACTOR
      DO 105 I=1,N
105 YY(I)=Y(I)+0.0001
C   COMPUTE THE WEIGHTING FACTOR NN FOR EACH BOUNDARY NODE
      DO 101 I=1,N
      NN(I)=0
      IF(Y(I).EQ.Y(I+1)) GO TO 102

```

```

        IF (YY(I).LT.Y(I+1).AND.YY(I).GT.Y(I)) NN(I)=NN(I)+1
        IF (I.EQ.1) GO TO 103
        IF (YY(I).LT.Y(I-1).AND.YY(I).GT.Y(I)) NN(I)=NN(I)+1
        GO TO 101
103 IF (YY(I).LT.Y(N).AND.YY(I).GT.Y(I)) NN(I)=NN(I)+1
        GO TO 101
102 NN(I)=1
101 CONTINUE
        DO 107 I=1,M
        NS(I) = 0
107 CONTINUE
        NA=0
        NRR=0
        XMN=XMAX-XMIN
        YMN=YMAX-YMIN
        DO 500 IK=1,NSET
        IR=0
        IL=0
C   GENERATE THE RANDOM NUMBER
        CALL RANDD(IX,IY,RDM)
        IX=IY
        XT=XMIN + RDM*XMN
        CALL RANDD(IA,IB,RDN)
        IA=IB
        YT=YMIN + RDN*YMN
        X(N+1)=X(1)
        Y(N+1)=Y(1)
C   CALCULATE THE NUMBER OF INTERSECTION ALONG THE X AXIS IN EITHER SIDE
        DO 300 K=1,N
        IF (YT.EQ.Y(K).AND.XT.EQ.X(K)) GO TO 310
        IF (YT.EQ.Y(K).AND.YT.EQ.Y(K+1)) GO TO 10
        IF (YT.EQ.Y(K)) GO TO 20
        IF (Y(K).GT.YT.AND.YT.GT.Y(K+1)) GO TO 40
        IF (YT.GT.Y(K).AND.Y(K+1).GT.YT) GO TO 40
        GO TO 300
10 IF (X(K).LT.XT.AND.X(K+1).GT.XT) GO TO 310
        IF (X(K).GT.XT.AND.X(K+1).LT.XT) GO TO 310
        IF (XT-X(K)) 11,310,12
11 IR=IR+NN(K)
        GO TO 300
12 IL=IL+NN(K)
        GO TO 300
20 IF (X(K)-XT) 12,310,11
40 XX=X(K) + (YT-Y(K))*(X(K+1)-X(K))/(Y(K+1)-Y(K))
        IF (XX-XT) 42,310,41
41 IR=IR+1
        GO TO 300
42 IL=IL+1
300 CONTINUE
C   CHECK WHETHER THE RANDOM POINT IS FALLING WITHIN THE BOUNDARY
        IF ((IR-IR/2*2).EQ.0.OR.(IL-IL/2*2).EQ.0) GO TO 302
310 NA=NA+1
C   ASSIGN THE FALLING WITHIN BOUNDARY POINT TO THE NEAREST STATION
        L(1)=(XT-AX(1))*2 + (YT-AY(1))*2

```



```

      SAVE=L(1)
      ISUB =1
      DO 91 I =2,M
      L(I)=(XT-AX(I))**2 +(YT-AY(I))**2
      IF(L(I)-SAVE)13,91,91
13   SAVE=L(I)
      ISUB=I
      91 CONTINUE
      NS(ISUB) =NS(ISUB)+1
      GO TO 599
302 NRP=NRR+1
599 CONTINUE
C   COMPUTE RELATIVE AREA RATIO AND WEIGHTING FACTOR OF EACH STATION
      AF= NA/(FLOAT(NA+NRR))
      AREA=AF*(XMAX-XMIN)*(YMAX-YMIN)*FACT
C   PRINT THE RESULTS
      WRITE(NW,7) AF,AREA
7   FORMAT(10X,21HRELATIVE AREA RATIO =,F9.6,24H   TOTAL AREA IN ACRES
1   =,F15.6//)
      DO 21 I=1,M
      WF(I) =NS(I)/FLOAT(NA)
21  WRITE(NW,6) I,WF(I)
6   FORMAT(10X,35HCOMPUTED WEIGHT OF RAINFALL STATION, 14.3H   =,F9.6//)
      GO TO 83
109 CALL ARAA(X,Y,NR,NW)
      GO TO 83
110 CALL MONTE(X,Y,YY,NN,NR,NW)
      GO TO 83
85  CALL EXIT
      END

```

```

      SUBROUTINE ARAA(X,Y,NR,NW)
      DIMENSION X(1),Y(1)
C   READ INPUT DATA
      READ(NR,6) N,FAC
6   FORMAT(I10,F10.3)
      READ(NR,4) (X(I),Y(I),I=1,N)
4   FORMAT(16F5.1)
      IFAC=FAC+0.01
      FACT=FAC*FAC/43560.0
      IF(IFAC.EQ.0) FACT=1.0
      WRITE(NW,108) N,FACT
108 FORMAT(/10X,29HNUMBER OF BOUNDARY SEGMENTS =,I4/10X,24HAREA CONVE
      RTING FACTOR =,F15.5//)
      SUM=0.0
      X(N+1)=X(1)
      Y(N+1)=Y(1)
      DO 7 K=1,N
      SUM1=(X(K+1)-X(K))*(Y(K+1)+Y(K))/2.0
7   SUM=SUM+SUM1
      SUM=SUM*FACT
      WRITE(NW,8) SUM

```

```

      R  FORMAT(/2X,21HTOTAL AREA IN ACRES =,F15.6//)
      RETURN
      END

```

```

      SUBROUTINE MONTE(X,Y,YY,NN,NR,NW)
      DIMENSION X(1),Y(1),YY(1),NN(1)
      IA=5
      IX=7

```

```

C  READ INPUT DATA
      READ(NR,8) N,NSET,XMIN,XMAX,YMIN,YMAX,FAC
      8  FORMAT(2I10,5F10.3)
      IFAC=FAC+0.01
      FACT=FAC*FAC/43560.0
      IF(IFAC.EQ.0) FACT=1.0
      WRITE(NW,108) N, NSET,XMIN,XMAX,YMIN,YMAX,FACT
108  FORMAT(/10X,29HNUMBER OF BOUNDARY SEGMENTS =,I4/10X,25HNUMBER OF
      1RANDOM POINTS =,I6/10X,16HX-AXIS MINIMUM =,F8.4/10X,16HY-AXIS MAXI
      2MUM =,F8.4/10X,16HY-AXIS MINIMUM =,F8.4/10X,16HY-AXIS MAXIMUM =,F8
      3.4/10X,24HAREA CONVERTING FACTOR =,F15.5//)
      READ(NR,9) (X(I),Y(I),I=1,N)
      9  FORMAT(16F5.1)
C  ADD A SMALL VALUE TO EACH NODE FOR OBTAINING THE NN FACTOR
      DO 106 I=1,N
106  YY(I)=Y(I)+0.00001
C  COMPUTE THE WEIGHTING FACTOR NN FOR EACH BOUNDARY NODE
      DO 101 I=1,N
      NN(I)=0
      IF(Y(I).EQ.Y(I+1)) GO TO 102
      IF(YY(I).LT.Y(I+1).AND.YY(I).GT.Y(I)) NN(I)=NN(I)+1
      IF(I.EQ.1) GO TO 103
      IF(YY(I).LT.Y(I-1).AND.YY(I).GT.Y(I)) NN(I)=NN(I)+1
      GO TO 101
103  IF(YY(I).LT.Y(N).AND.YY(I).GT.Y(I)) NN(I)=NN(I)+1
      GO TO 101
102  NN(I)=1
101  CONTINUE
C  GENERATE THE RANDOM POINT
      NA=0
      NRR=0
      XMN=XMAX-XMIN
      YMN=YMAX-YMIN
      DO 500 IK=1,NSET
      IR=0
      IL=0
      CALL RAND0(IX,IY,RDM)
      IX=IY
      XT=XMIN + RDM*XMN
      CALL RAND0(IA,IB,RDN)
      IA=IB
      YT=YMIN + RDN*YMN
      X(N+1)=X(1)
      Y(N+1)=Y(1)

```

```

C  CALCULATE THE NUMBER OF INTERSECTION ALONG THE X AXIS
  DO 300 K=1,N
    IF(YT.EQ.Y(K).AND.XT.EQ.X(K)) GO TO 310
    IF(YT.EQ.Y(K).AND.YT.EQ.Y(K+1)) GO TO 10
    IF(YT.EQ.Y(K)) GO TO 20
    IF(Y(K).GT.YT.AND.YT.GT.Y(K+1)) GO TO 40
    IF(YT.GT.Y(K).AND.Y(K+1).GT.YT) GO TO 40
    GO TO 300
10  IF(X(K).LT.XT.AND.X(K+1).GT.XT) GO TO 310
    IF(X(K).GT.XT.AND.X(K+1).LT.XT) GO TO 310
    IF(XT-X(K)) 11,310,12
11  IR=IR+NN(K)
    GO TO 300
12  IL=IL+NN(K)
    GO TO 300
20  IF(X(K)-XT) 12,310,11
40  XX=X(K)+ (YT-Y(K))*(X(K+1)-X(K))/(Y(K+1)-Y(K))
    IF(XX-XT) 42,310,41
41  IR=IR+1
    GO TO 300
42  IL=IL+1
300 CONTINUE
C  CHECK WHETHER THE RANDOM POINT IS FALLING WITHIN THE BOUNDARY
    IF((IR-IR/2*2).EQ.0.OR.(IL-IL/2*2).EQ.0) GO TO 302
310 NA=NA+1
    GO TO 500
302 NRR=NRR+1
500 CONTINUE
    AF= NA/(FLOAT(NA+NRR))
    AREA=AF*(XMAX-XMIN)*(YMAX-YMIN)*FACT
    WRITE(NW,7) AF,AREA
7  FORMAT(10X,21HRELATIVE AREA RATIO =,F9.6,24H  TOTAL AREA IN ACRES
1  =,F15.6//)
    RETURN
    END

```

```

C  SURROUTINE RANDD(IX,IY,YFL)
  GENERATE THE RANDOM NUMBER
  IY=IX*4099
  IF(IY)5,6,6
5  IY=IY+8388607+1
6  YFL=IY
  YFL=YFL/8388607.0
  RETURN
  END

```